

Direct measurement of a pure spin current by a polarized light beam*

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The photon helicity may be mapped to a spin-1/2, whereby we put forward an intrinsic interaction between a polarized light beam as a “photon spin current” and a pure spin current in a semiconductor, which arises from the spin-orbit coupling in valence bands as a pure relativity effect without involving the Rashba or the Dresselhaus effect due to inversion asymmetries. The interaction leads to circular optical birefringence, which is similar to the Faraday rotation in magneto-optics but nevertheless involve no net magnetization. The birefringence effect provide a direct, non-demolition measurement of pure spin currents.

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Measuring spin currents is an indispensable part of spin-based electronics [1]. Spin-polarized currents flow with net magnetization and therefore can be sensed by conventional Faraday or Kerr rotation [2, 3, 4] or through ferromagnetic filters [5, 6]. A pure spin current consists of two counter-flowing currents with equal amplitude but opposite spin polarizations and thus bears neither net charge current nor net magnetization. Pure spin currents have been inferred from a few pioneering experiments, being converted either into charge/voltage signals [7, 8, 9] via the spin Hall effect [10, 11, 12, 13], or into spin-polarized electrons or excitons where the currents terminate [14, 15, 16, 17]. An open and fundamental question is: How can a pure spin current be directly measured?

A clue to the answer is the Ampère effect where a charge current, even with net charge density being neutral everywhere, can be detected by other moving charges or another current. A straightforward analogue suggests that a pure spin current could be measured by another spin current. In general, for a current breaking the intrinsic symmetry of a system, a phenomenological coupling can be constructed with another current of the same symmetry-breaking type. A pure spin current conserves the time-inversion symmetry but breaks the rotation and space-inversion symmetries, and can be formulated as a rank-2 pseudo-tensor

$$\mathbb{J} = J_X \mathbf{X}\mathbf{X} + J_Y \mathbf{Y}\mathbf{Y} + J_Z \mathbf{Z}\mathbf{Z} \equiv \mathbf{J}\mathbf{Z}, \quad (1)$$

where \mathbf{Z} is the current direction, \mathbf{X} and \mathbf{Y} are the transverse directions [see Fig. 1 (a)], and \mathbf{J} is along the spin polarization direction. To form a phenomenological current-current coupling in a system of all the fundamental symmetries, the

probe current should be of the same tensor type. In lieu of a real spin current for the probe, we would rather use a polarized light beam which is more feasible. A light can be regarded as a “photon spin current” [18] by mapping the photon polarization into a spin-1/2 with the Jones vector representation [19], $\cos \frac{\theta}{2} e^{i\phi/2} \mathbf{n}_+ + \sin \frac{\theta}{2} e^{-i\phi/2} \mathbf{n}_- \sim |\theta, \phi\rangle$, where the right/left circular polarization \mathbf{n}_{\pm} corresponds to the spin up/down state $|\uparrow / \downarrow\rangle$ quantized along the light propagation direction. The “spin current” tensor for a light with electric field $\mathbf{F}(\mathbf{r}, t) = (F_+ \mathbf{n}_+ + F_- \mathbf{n}_-) e^{i\mathbf{q}\cdot\mathbf{r} - i\omega_q t} + \text{c.c.}$ can be formulated as

$$\mathbb{I} \equiv q(I_x \mathbf{x}\mathbf{z} + I_y \mathbf{y}\mathbf{z} + I_z \mathbf{z}\mathbf{z}) \equiv q\mathbf{I}\mathbf{z}, \quad (2a)$$

$$I_j = \frac{1}{2} \sum_{\mu, \nu=\pm} \sigma_{\mu\nu}^j F_{\mu}^* F_{\nu}, \quad (2b)$$

where σ^j ($j = x, y, z$) is the Pauli matrix, and the unit axis vectors \mathbf{x} , \mathbf{y} , and \mathbf{z} are defined by $\mathbf{n}_{\pm} \equiv (\mp \mathbf{x} - i\mathbf{y}) / \sqrt{2}$ and $\mathbf{z} \equiv \mathbf{q}/q$.

The specific form of the phenomenological coupling depends on the microscopic mechanisms. Consider an n-doped bulk III-V compound semiconductor with a direct band gap (such as GaAs) and a pure spin current due to a steady non-equilibrium distribution $\hat{\rho}$ [see Fig. 1 (b)]. An effective coupling between a polarized beam and the spin current can be mediated by the interband virtual excitations. Since the light polarization essentially couples only to the orbital motion of electrons, spin-orbit interaction is required to establish the effective coupling. As there is inherent spin-orbit coupling in the valence bands as a relativity effect, the Rashba or Dresselhaus effect due to inversion asymmetries [20, 21, 22] is not a necessity.

Assuming the light is tuned below the Fermi surface and near the band edge, we consider optical transitions between the conduction band (CB) and the heavy-hole (HH) and light-hole (LH) bands, and neglect the split-off band (SO) [see Fig. 1 (b)]. The Luttinger-Kohn Hamiltonian h_{LK} for the valence bands near the band edge is [23]

$$h_{\text{LK}} = \frac{\hbar^2}{2m} \left[\left(\gamma_1 + \frac{5}{2} \gamma_2 \right) \nabla^2 - 2\gamma_2 (\nabla \cdot \mathbf{K})^2 \right] \quad (3)$$

*This is a corrected version of Phys. Rev. Lett. **100**, 086603 (2008), where the interband transition Hamiltonian in Eq. (5) contains errors in the spin notations which leads to errors in the effective coupling Hamiltonian in Eqs. (7) and (10). Unchanged is the main conclusion that a pure spin current can be detected by a polarized light beam through a current-current coupling, but the Voigt effect now is absent and the Faraday rotation is enhanced and has different dependence on the incident angles.

where \mathbf{K} is a spin-3/2 for the total angular momentum of a hole. We have for simplicity neglected the valence-band anisotropy which would not change the essential results in this paper. The energy dispersion for a hole with the magnetic quantum number K_p quantized along the momentum \mathbf{p} is $E_{K_p}(\mathbf{p}) = [\gamma_1 + (5/2 - 2K_p^2)\gamma_2]\hbar^2 p^2 / (2m)$. The non-interacting Hamiltonian for electrons and holes is

$$\hat{H}_0 = \sum_{\pm, \mathbf{p}} (E_{e\mathbf{p}} \hat{e}_{\pm\mathbf{p}}^\dagger \hat{e}_{\pm\mathbf{p}} + E_{h\mathbf{p}} \hat{h}_{\pm\mathbf{p}}^\dagger \hat{h}_{\pm\mathbf{p}} + E_{l\mathbf{p}} \hat{l}_{\pm\mathbf{p}}^\dagger \hat{l}_{\pm\mathbf{p}}), \quad (4)$$

where $\hat{e}_{\pm\mathbf{p}}$ annihilates an electron with spin $\pm 1/2$, $\hat{h}_{\pm\mathbf{p}}$ and $\hat{l}_{\pm\mathbf{p}}$ annihilate heavy and light holes with $K_p = \pm 3/2$ and $\pm 1/2$, respectively, $E_{h\mathbf{p}} = E_{\pm 3/2}(\mathbf{p}) = \hbar^2 p^2 / (2m_h)$, $E_{l\mathbf{p}} = E_{\pm 1/2}(\mathbf{p}) = \hbar^2 p^2 / (2m_l)$, and $E_{e\mathbf{p}} = \hbar^2 p^2 / (2m_e)$. Here m_e , m_h , and m_l are in turn the electron, HH, and LH effective mass. Notice that the angular momentum \mathbf{K} is quantized along \mathbf{p} so that the spin-orbit coupling in the valence bands is automatically included. There is no Rashba effect in the bulk system, and the Dresselhaus spin splitting is negligible [24].

The interband transition has the Hamiltonian

$$\begin{aligned} \hat{H}_1 = & d_{cv}^* \sum_{\mu, \nu, \mathbf{p}} F_{\nu}^* \mathbf{n}_{\nu}^* \cdot (\mathbf{n}_{\mu, \mathbf{p}} \hat{h}_{\mu-\mathbf{p}} \hat{e}_{\mu\mathbf{q}+\mathbf{p}} + \\ & \frac{1}{\sqrt{3}} \mathbf{n}_{\mu, \mathbf{p}} \hat{l}_{\mu-\mathbf{p}} \hat{e}_{\mu\mathbf{q}+\mathbf{p}} - \sqrt{\frac{2}{3}} \mathbf{z}_{\mathbf{p}} \hat{l}_{\mu-\mathbf{p}} \hat{e}_{\mu\mathbf{q}+\mathbf{p}}) + \text{h.c.}, \end{aligned} \quad (5)$$

with $\mathbf{n}_{\pm, \mathbf{p}}$ denoting the right/left circular polarization about \mathbf{p} which are defined as $\mathbf{n}_{\pm, \mathbf{p}} \equiv (\mp \mathbf{x}_{\mathbf{p}} - i \mathbf{y}_{\mathbf{p}}) / \sqrt{2}$, $\mathbf{z}_{\mathbf{p}} \equiv \mathbf{p} / p$, and $\bar{\mu} \equiv -\mu$.

The effective Hamiltonian between the light beam and the spin current is obtained by the second-order perturbation as

$$\mathcal{H}_{\text{eff}} = \text{Tr} \left[\hat{\rho} \hat{H}_1 (\hbar\omega_q - \hat{H}_0)^{-1} \hat{H}_1 \right], \quad (6)$$

and explicitly worked out to be

$$\mathcal{H}_{\text{eff}} = -|d_{cv}|^2 \sum_{\sigma, \sigma'} F_{\sigma'}^* F_{\sigma} \mathbf{n}_{\sigma'} \mathbf{n}_{\sigma}^* : \quad (7a)$$

$$\sum_{\mu, \mathbf{p}} \left[f_{\bar{\mu}\mathbf{p}\bar{\mu}\mathbf{p}, \mathbf{q}+\mathbf{p}} \frac{\mathbf{n}_{\mu, \mathbf{p}} \mathbf{n}_{\mu, \mathbf{p}}^*}{E_{e\mathbf{q}+\mathbf{p}} + E_{h-\mathbf{p}} - \hbar\omega_q} \right] \quad (7b)$$

$$+ \frac{1}{3} f_{\bar{\mu}\mathbf{p}\bar{\mu}\mathbf{p}, \mathbf{q}+\mathbf{p}} \frac{\mathbf{n}_{\bar{\mu}, \mathbf{p}} \mathbf{n}_{\bar{\mu}, \mathbf{p}}^* + 2\mathbf{z}_{\mathbf{p}} \mathbf{z}_{\mathbf{p}}^*}{E_{e\mathbf{q}+\mathbf{p}} + E_{l-\mathbf{p}} - \hbar\omega_q} \quad (7c)$$

$$- \frac{\sqrt{2}}{3} f_{\mu\mathbf{p}\bar{\mu}\mathbf{p}, \mathbf{q}+\mathbf{p}} \frac{\mathbf{n}_{\bar{\mu}, \mathbf{p}} \mathbf{z}_{\mathbf{p}}^* + \mathbf{z}_{\mathbf{p}} \mathbf{n}_{\bar{\mu}, \mathbf{p}}^*}{E_{e\mathbf{q}+\mathbf{p}} + E_{l-\mathbf{p}} - \hbar\omega_q} \Big], \quad (7d)$$

where $f_{\mu\nu, \mathbf{p}} \equiv \text{Tr} [\hat{\rho} \hat{e}_{\mu\mathbf{p}}^\dagger \hat{e}_{\nu\mathbf{p}}]$, and $\mu_{\mathbf{p}}$ indicates that the spin is quantized along \mathbf{p} . Here we have omitted the trivial background constant. The physical processes for different terms are identified as follow: (7b) accounts for the HH-CB transitions where a (virtually) absorbed photon has to be emitted with the same circular polarization and thus the electron spin is conserved. A LH state contains both circular and linear orbital components, so a LH-CB transition can be either circularly or linearly polarized. In (7c), the LH-CB transitions

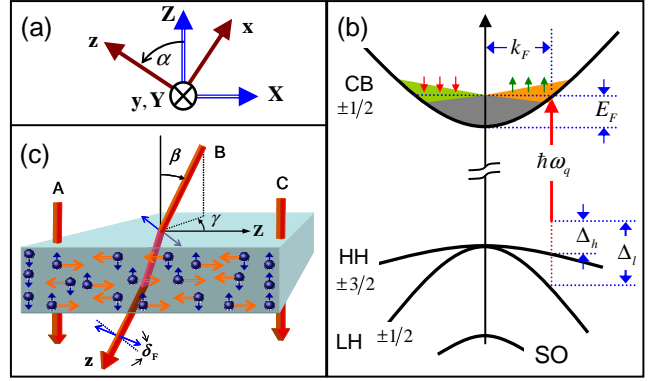


FIG. 1: (color online) (a) The spin current direction (\mathbf{Z}), the light beam direction (\mathbf{z}), and the associated coordinate systems. Without loss of generality, \mathbf{y} and \mathbf{Y} are set parallel. (b) Schematic band structure near the Γ point of an n-doped III-V compound semiconductor, and illustration of a non-equilibrium electron distribution for a pure spin current. (c) Setup for measuring a transverse pure spin current. Case A and C show the measurement of spins accumulated at the edges by the Faraday rotation of a normal incident light as in Ref. [14]. Case B shows the direct measurement of the spin current in the middle region by the Faraday rotation of an oblique light beam.

in virtual absorption and emission have the same polarization and in turn the electron spin is conserved. In (7d), the transitions from and to the LH bands have different polarizations, leading to orbital angular momentum transfer between the light and the electrons, while the total angular momentum is still conserved.

As we are not interested in charge effects such as a charge current, we shall drop spin-independent populations like $f_{++\mathbf{p}}$ and $f_{--\mathbf{p}}$ (which contributes only a change of the background refraction index) but keep only the spin distribution $\mathbf{s}(\mathbf{p}) \equiv \frac{1}{2} \sum_{\mu, \nu} \sigma_{\mu, \nu} f_{\mu\nu, \mathbf{p}}$. Furthermore, we consider a pure spin current and assume the net spin polarization of the system is zero. The spin current is defined by the dyadic of the spin polarization and the velocity, summed over the momentum space:

$$\mathbb{J} \equiv e \sum_{\mathbf{p}} \mathbf{s}(\mathbf{p}) \hbar^{-1} \nabla_{\mathbf{p}} E_{e\mathbf{p}}. \quad (8)$$

A pure spin current results when the electrons of opposite velocities have opposite spins. With the charge background dropped and the total spin polarization absent, the leading contribution to the coupling in Eq. (7) would come from the spin current as discussed below.

To proceed from Eq. (7), let us first neglect the small light wavevector \mathbf{q} . Since the light excites the same electron spin for opposite electron momenta $\pm\mathbf{p}$, the polarized light is only coupled to a net spin polarization, but not to a pure spin current.

Then we include the small light wavevector \mathbf{q} up to its first order by the expansion $E_{e\mathbf{q}+\mathbf{p}} \approx E_{e\mathbf{p}} + \mathbf{q} \cdot \nabla_{\mathbf{p}} E_{e\mathbf{p}}$ and $f_{\mu, \nu}(\mathbf{q} + \mathbf{p}) \approx f_{\mu, \nu}(\mathbf{p}) + \mathbf{q} \cdot \nabla_{\mathbf{p}} f_{\mu, \nu}(\mathbf{p})$. The gradient in the mo-

momentum space $\nabla_{\mathbf{p}}$ contributes the electron velocity which is opposite for opposite momenta. So the light couples to opposite electron spin for opposite momenta, and in turn to a pure spin current.

To determine the specific form of the effective Hamiltonian, we assume that the electron distribution deviates only slightly from the equilibrium with Fermi wavevector k_F [or Fermi energy E_F , see Fig. 1 (b)] and the spin distribution has the form

$$\mathbf{s}(\mathbf{p}) = s(p) \cos \theta_{\mathbf{p}}, \quad (9)$$

where $\theta_{\mathbf{p}}$ is the angle between \mathbf{p} and the current direction \mathbf{Z} . Such a distribution is usually the case for weak currents. The light frequency is lower by Δ_h and Δ_l than the transition energy from the HH and the LH bands to the Fermi surface, respectively. A straightforward integration over the momentum space yields the effective coupling as

$$\mathcal{H}_{\text{eff}} = \zeta_1 q I_z \mathbf{z} \cdot \mathbb{J} \cdot \mathbf{z} + \zeta_2 q I_z J_z, \quad (10)$$

with

$$\begin{aligned} \zeta_1 &\equiv \frac{\hbar |d_{\text{cv}}|^2}{e} \left(\frac{4m_e}{5\Delta_h^2 m_h} + \frac{8m_e}{15\Delta_l^2 m_l} - \frac{1}{5\Delta_h E_F} + \frac{1}{5\Delta_l E_F} \right), \\ \zeta_2 &\equiv \frac{\hbar |d_{\text{cv}}|^2}{e} \left(\frac{2m_e}{5\Delta_h^2 m_h} - \frac{2m_e}{5\Delta_l^2 m_l} - \frac{3}{5\Delta_h E_F} + \frac{3}{5\Delta_l E_F} \right). \end{aligned}$$

For a spin distribution different from Eq. (9), the coupling constants shown above will only be quantitatively changed in some form factors. Obviously, the effective coupling, being a tensor contraction between the pure spin current and the “photon spin current”, has all the fundamental symmetries of the system.

The linear optical susceptibility is directly related to the phenomenological Hamiltonian by

$$\chi_{\mu,\nu} + \chi_{\nu,\mu}^* = \frac{1}{\epsilon_0} \frac{\partial^2 \mathcal{H}_{\text{eff}}}{\partial F_{\mu}^* \partial F_{\nu}}. \quad (11)$$

Here ϵ_0 is the vacuum permittivity. Since the light is tuned off-resonant from the available transitions, no real absorption will occur, but different phaseshifts for different light polarizations would be induced. In other words, a pure spin current would produce circular birefringence effect, which is similar to the Faraday rotation in magnetooptics [25], but nonetheless involves no net magnetization.

The two terms in Eq. (10) depends on the small light wavevector q , characteristic of coupling to the magnetic dipole moment of a chiral quantity. Indeed, both J_z and $\mathbf{z} \cdot \mathbb{J} \cdot \mathbf{z}$ are longitudinal components of the spin current in which the current propagation and polarization are parallel, and hence are chiral. The chiral spin current is coupled to a circularly polarized photon current, I_z , which is also a chiral quantity. These q -dependent terms, stemming from total angular momentum conserving processes [(7b), (7c) and (7d)], would not cause the circular polarizations of a light to be flipped but

rather induce a polarization-dependent phaseshift. The susceptibility for opposite circular polarizations are opposite:

$$\chi_{++} = -\chi_{--} = \frac{1}{4\epsilon_0} q (\zeta_1 \mathbf{z} \cdot \mathbb{J} \mathbf{z} \cdot \mathbf{z} + \zeta_2 J_z), \quad (12)$$

which results in circular birefringence similar to the Faraday rotation [25] but with no net magnetization. The Faraday rotation angle is

$$\delta_F = \omega_q L (\chi_{++} - \chi_{--}) / (4nc). \quad (13)$$

where L is the light propagation distance, n is the material refractive index, c is the light velocity in vacuum.

To demonstrate the feasibility of using the optical birefringence for direct measurement of a pure spin current, we consider a realistic case that was studied in Ref. [14] where a transverse spin current is caused by the spin Hall effect. In Ref. [14], the Faraday rotation of a light normal to the sample surface (and parallel to the spin polarization) is measured, with non-vanishing results only near the edges where spins are accumulated [case A and C in Fig. 1 (c)]. The absence of Faraday rotation in the middle region where the spin current flows with no net spin polarization is readily explained by Eq. (12): In the experimental setup, $\mathbf{Z} \cdot \mathbf{z} = 0$ and $J_z = 0$. To directly measure the spin current where it flows, we propose to tilt the light beam with a zenith angle β from the normal direction and an azimuth angle γ from the spin current direction and detect the Faraday rotation δ_F in the middle region [Case B in Fig. 1 (c)]. The results are predicted to vary with the incident angles as

$$\delta_F(\beta, \gamma) = \delta_{F,0} \sin \beta \cos \gamma. \quad (14)$$

To estimate the amplitudes of the effect, we use the parameters for the GaAs sample in Ref. [14], i.e., $m_e = 0.067m_0$ (m_0 being the free electron mass), $m_h = 0.45m_0$, $m_l = 0.082m_0$, $L = 2.0 \mu\text{m}$, $n = 3.0$, $d_{\text{cv}} = 6.7 e\text{\AA}$, and $E_F = 5.3 \text{ meV}$ ($k_F = 0.96 \times 10^6 \text{ cm}^{-1}$ for doping density $3 \times 10^{16} \text{ cm}^{-3}$). We take the light wavelength to be around 800 nm, and the detuning $\Delta_h = 1.0 \text{ meV}$ and $\Delta_l = 4.5 \text{ meV}$. For a spin current with amplitude $J_s = 20 \text{ nA}\mu\text{m}^{-2}$ [14], the maximum Faraday rotation, reached when $\beta \rightarrow \pi/2$ and $\gamma \rightarrow 0$, is $\delta_{F,0} \approx 0.38 \mu\text{rad}$, measurable in experiments [14].

In conclusion, the intrinsic interaction between a polarized light and a spin current may induce measurable circular birefringence as a direct non-demolition measurement of a pure spin current. Unlike the generation of spin currents [8, 9, 26, 27, 28, 29], the measurement scheme proposed here does not rely on the inversion asymmetry of the system.

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